206

1) ii)

$$L.H.S = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a-b & c(b-a) \\ 0 & b-c & a(c-b) \\ 1 & c & ab \end{vmatrix} \quad [R_1' = R_1 - R_2 \ \& \ R$$

$$= \begin{vmatrix} 0 & a-b & -c(a-b) \\ 0 & b-c & -a(b-c) \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} a-b & -c(a-b) \\ b-c & -a(b-c) \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 1 & -c \\ 1 & -a \end{vmatrix}$$

$$= (a-b)(b-c)(-a+c)$$

$$= (a-b)(b-c)(c-a) \text{ [proved]}$$

$= R.H.S$

iii) $\text{L.H.S} = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

$$= abc \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= abc \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix} \quad \left[\begin{matrix} C_1' = C_1 - C_2 \\ C_2' = C_2 - C_3 \end{matrix}\right]$$

$$= abc \begin{vmatrix} a-b & b-c \\ a^2-b^2 & b^2-c^2 \end{vmatrix}$$

$$= abc \begin{vmatrix} a-b & b-c \\ (a+b)(a-b) & (b+c)(b-c) \end{vmatrix}$$

$$= abc\,(a-b)(b-c) \begin{vmatrix} 1 & 1 \\ a+b & b+c \end{vmatrix}$$

$$= abc(a-b)(b-c)(\cancel{b}+c-a-\cancel{b})$$

$$= abc\,(a-b)(b-c)(c-a) \text{ [proved]}$$

Scanned with CamScanner

iv)

$$L.H.S = \begin{vmatrix} 1 & 1 & 1 \\ a & a^2 & a^3 \\ b & b^2 & b^3 \end{vmatrix}$$

$$= ab \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix}$$

$$= ab \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & a^2-1 \\ 1 & b & b^2 \end{vmatrix} \quad [R_2' = R_2 - R_1]$$

$$= ab \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & a^2-1 \\ 0 & b-1 & b^2-1 \end{vmatrix} \quad [R_3' = R_3 - R_1]$$

$$= ab \begin{vmatrix} a-1 & a^2-1 \\ b-1 & b^2-1 \end{vmatrix}$$

$$= ab \begin{vmatrix} a-1 & (a+1)(a-1) \\ b-1 & (b+1)(b-1) \end{vmatrix}$$

$$= ab(a-1)(b-1) \begin{vmatrix} 1 & a+1 \\ 1 & b+1 \end{vmatrix}$$

$$= ab(a-1)(b-1)(b+\cancel{1}-a-\cancel{1})$$

$$= ab(a-1)(b-1)(b-a) \text{ [proved]}$$

v)

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^3 & y^3-x^3 & z^3-x^3 \end{vmatrix} \quad \left[\begin{array}{l} C_2' = C_2 - C_1 \\ \& \; C_3' = C_3 - C_1 \end{array} \right]$$

$$= \begin{vmatrix} y-x & z-x \\ (y-x)(y^2+xy+x^2) & (z-x)(z^2+xz+x^2) \end{vmatrix}$$

$$= (y-x)(z-x) \begin{vmatrix} 1 & 1 \\ y^2+xy+x^2 & z^2+xz+x^2 \end{vmatrix}$$

$$= -(x-y)(z-x)(z^2+xz+\cancel{x^2}-y^2-xy-\cancel{x^2})$$

$$= -(x-y)(z-x)\left[(z+y)(z-y)+x(z-y)\right]$$

$$= \cancel{-}(x-y)(z-x)(z-y)(x+y+z)$$

$$= (x-y)(y-z)(z-x)(x+y+z)$$

[proved]

Scanned with CamScanner

1) $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$

$= \begin{vmatrix} a-b & a^2-b^2 & bc-ca \\ b-c & b^2-c^2 & ca-ab \\ c & c^2 & ab \end{vmatrix} \quad \left[\begin{matrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \end{matrix}\right]$

$= (a-b)(b-c) \begin{vmatrix} 1 & a+b & -c \\ 1 & b+c & -a \\ c & c^2 & ab \end{vmatrix}$

$= (a-b)(b-c) \begin{vmatrix} 1 & a+b & -c \\ 0 & c-a & c-a \\ c & c^2 & ab \end{vmatrix} \quad [R_2' = R_2 - R_1]$

$= (a-b)(b-c)(c-a) \begin{vmatrix} 1 & a+b & -c \\ 0 & 1 & 1 \\ c & c^2 & ab \end{vmatrix}$

$= (a-b)(b-c)(c-a) \begin{vmatrix} 1 & a+b+c & -c \\ 0 & 0 & 1 \\ c & c^2-ab & ab \end{vmatrix} \quad [C_2' = C_2 - C_3]$

$= (a-b)(b-c)(c-a)(-1) \begin{vmatrix} 1 & a+b+c \\ c & c^2-ab \end{vmatrix}$

$$= (a-b)(b-c)(c-a)(-1)[c^2 - ab - ac - bc - c^2]$$

$$= (a-b)(b-c)(c-a)(-1)(ab+bc+ca)$$

vii) $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

$$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix} \quad [C_1' = C_1 + C_2 + C_3]$$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

$$= a+b+c \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} b-c & c-a \\ c+a-2b & a+b-2c \end{vmatrix} \quad [R_3' = R_3 - 2R_1]$$

$$= (a+b+c) \begin{vmatrix} b-c & c-a \\ a-b & b-c \end{vmatrix} \quad [R_2' = R_2 + R_1]$$

$$= (a+b+c)\left[(b-c)^2 - (a-b)(c-a)\right]$$

$$= (a+b+c)\left[b^2 - 2bc + c^2 - ab + a^2 + bc - ab\right]$$

$$= (a+b+c)\left[a^2 + b^2 + c^2 - ab - bc - ca\right]$$

$$= a^3 + b^3 + c^3 - 3abc \text{ [proved]}.$$

2.
i)
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= \begin{vmatrix} 3+a & 1 & 1 \\ 3+a & 1+a & 1 \\ 3+a & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 0 & 0 & 1 \\ -a & a & 1 \\ 0 & -a & 1+a \end{vmatrix} \quad \left[\begin{aligned} C_1' &= C_1 - C_2 \\ C_2' &= C_2 - C_3 \end{aligned}\right]$$

$$= (3+a)\begin{vmatrix} -a & a \\ 0 & -a \end{vmatrix}$$

$$= (3+a)a^2 \text{ [proved]}.$$